LINEAR PREDICTION

- The orthogonality principle and optimal filtering
- The orthogonality principle for linear prediction
- The autoregressive (AR) model
- Backward prediction and the anticausal AR model
- The Levinson Recursion
- Lattice representation of filters

LINEAR PREDICTION (cont'd.)

- Partial correlation in the context of linear prediction
- Minimum-phase property of the prediction error filter
- The Schur algorithm
- "Split" algorithms
- Relations to triangular decomposition
- Lattice form for Wiener filter

LINEAR MEAN-SQUARE ESTIMATION

FORM OF ESTIMATE

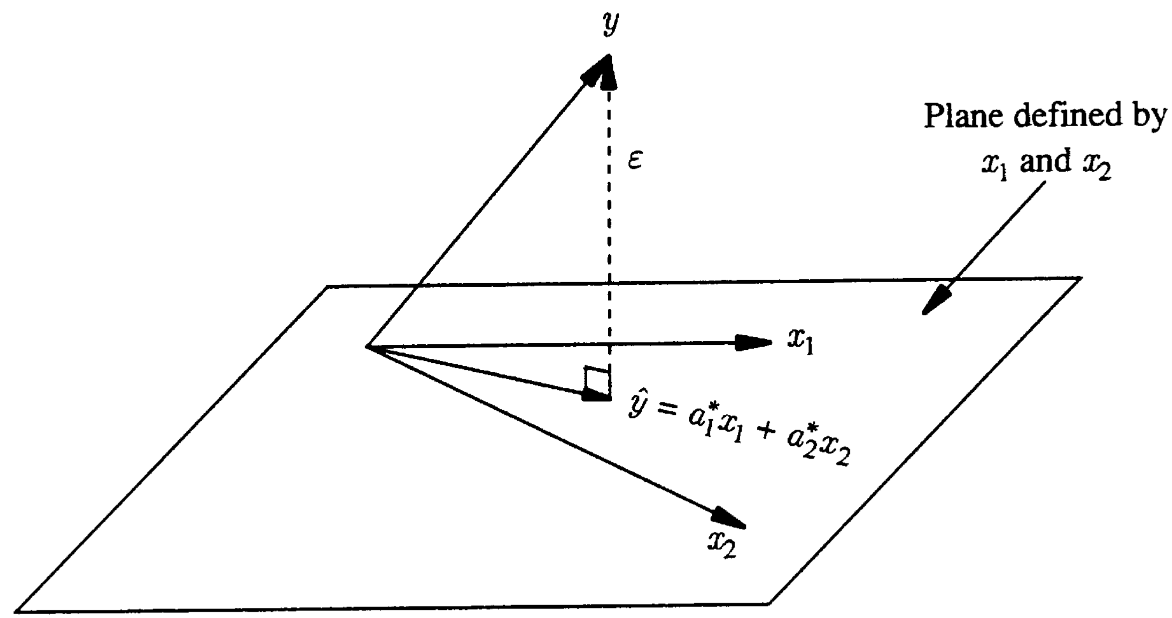
$$\hat{y} = \mathbf{a}^{*T}\boldsymbol{x} = a_1^* x_1 + a_2^* x_2 + \ldots + a_N^* x_N$$

ORTHOGONALITY PRINCIPLE

Theorem: Let $\varepsilon = y - \hat{y}$ be the error in estimation. Then $\mathbf{a}$ minimizes the mean-square error $\sigma_\varepsilon^2 = \mathcal{E}\left\{|y - \hat{y}|^2\right\}$ if $\mathbf{a}$ is chosen such that $\mathcal{E}\{x_i \varepsilon^*\} = \mathcal{E}\left\{\varepsilon x_i^*\right\} = 0 \quad i = 1, 2, \ldots, N$, that is, if the error is <u>orthogonal</u> to the observations. Further the minimum mean-square error is given by $\sigma_\varepsilon^2 = \mathcal{E}\{y\varepsilon^*\} = \mathcal{E}\{\varepsilon y^*\}$.

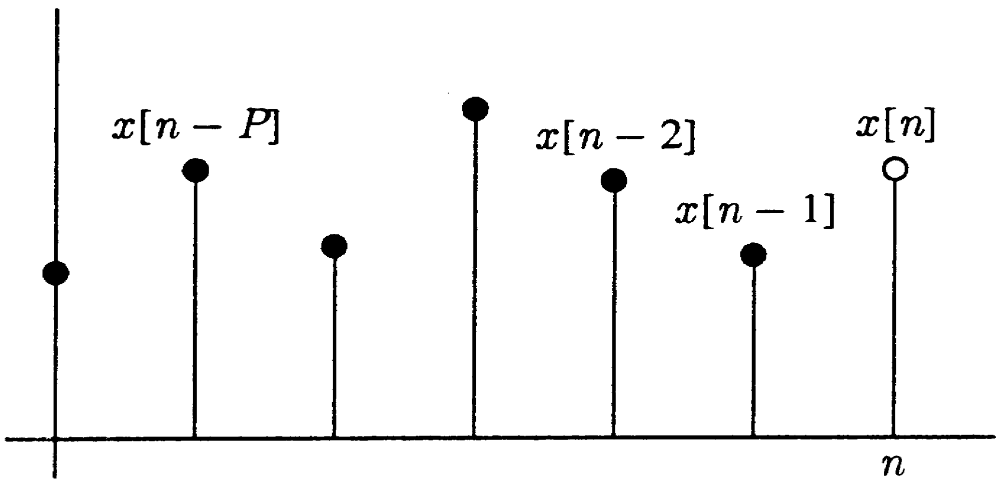
VECTOR SPACE INTERPRETATION OF THE ORTHOGONALITY PRINCIPLE

- Elements are random variables.
- Inner product is expectation.



THE LINEAR PREDICTION PROBLEM

Estimate the present value of a signal $x[n]$ from past values $x[n-1], x[n-2], \ldots, x[n-P]$.



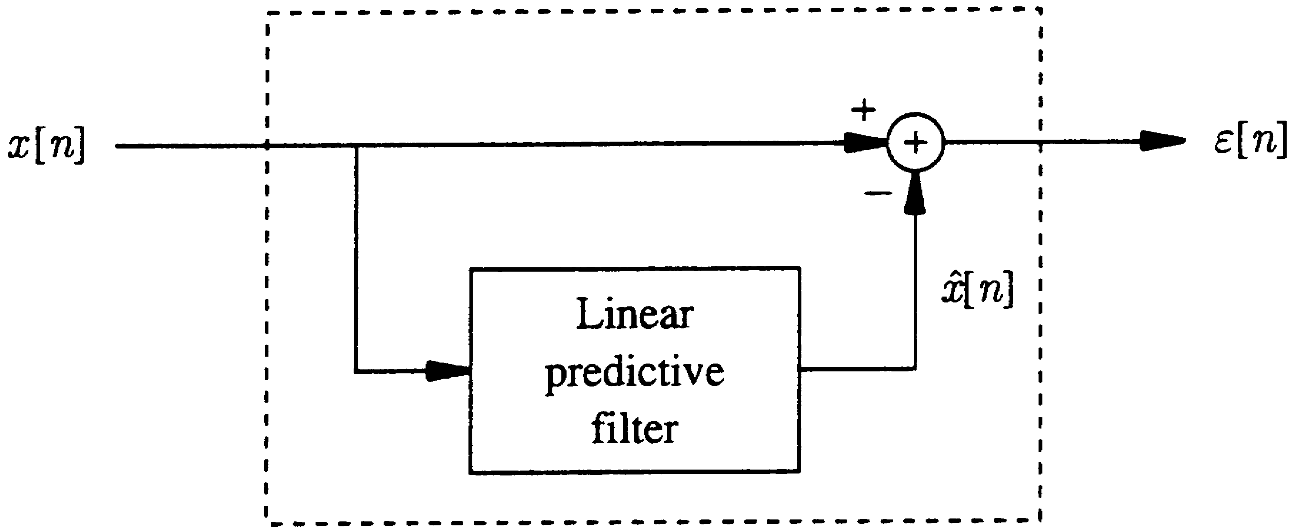
Minimize:

$$\sigma_\varepsilon^2 = \mathcal{E}\left\{|\varepsilon[n]|^2\right\}$$

$$\varepsilon[n] = x[n] - \hat{x}[n]$$

$$\hat{x}[n] = -a_1^* x[n-1] - a_2^* x[n-2] - \cdots - a_P^* x[n-P]$$

PREDICTION ERROR FILTER



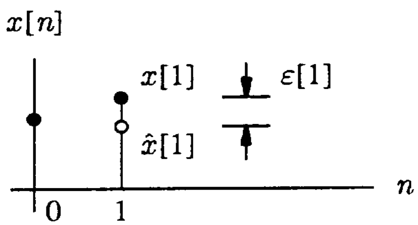
Prediction error filter

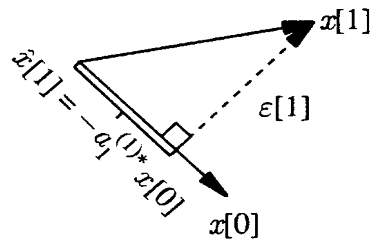
$$\varepsilon[n] = x[n] - \widehat{x}[n] = \sum_{k=0}^{P} a_k^* x[n-k]\,; \qquad \text{with } a_0 \equiv 1$$

ORTHOGONALITY PRINCIPLE FOR LINEAR PREDICTION

Theorem: Let $\varepsilon[n] = x[n] - \widehat{x}[n]$ be the error in estimation. Then the prediction error filter with coefficients $1, a_1^*, a_2^*, \ldots, a_P^*$ minimizes the prediction error variance σ_ε^2 if the filter coefficients are chosen such that $\mathcal{E}\{x[n-i]\varepsilon^*[n]\} = 0 \quad i = 1, 2, \ldots, P$, that is, if the error is orthogonal to the observations. Further the (minimum) prediction error variance is given by $\sigma_\varepsilon^2 = \mathcal{E}\{x[n]\varepsilon^*[n]\}$.

VECTOR SPACE INTERPRETATION OF LINEAR PREDICTION





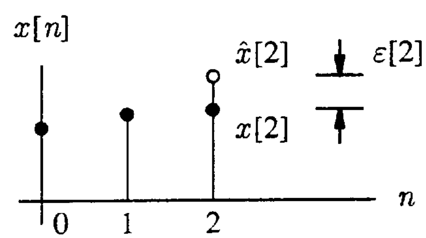
First order

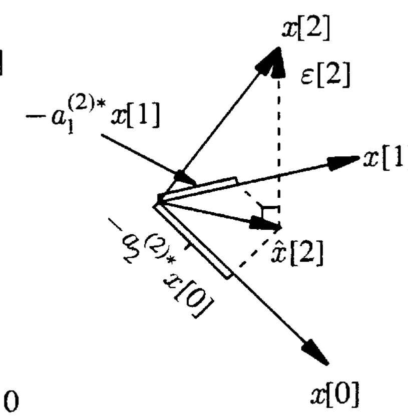
$\hat{x}[1] = -a_1^{(1)*}x[0]$

$x[1] = \hat{x}[1] + \varepsilon[1]$

$\mathcal{E}[\varepsilon[1]x[0]] = 0$

Second order





$\hat{x}[2] = -a_1^{(2)*}x[1] - a_2^{(2)*}x[0]$

$x[2] = \hat{x}[2] + \varepsilon[2]$

$\mathcal{E}[\varepsilon[2]x[1]] = 0\ ;\ \mathcal{E}[\varepsilon[2]x[0]] = 0$

WHITENING BY LINEAR PREDICTION

- Error sequence orthogonal, $\mathcal{E}\{\varepsilon[i]\varepsilon^*[j]\} = 0$.
- Prediction error filter whitens the process.